

## Problem Set 11

Harvard SEAS - Fall 2021

Due: Sun Dec 5, 2021 (11:59pm)

**Your name:****Collaborators:****No. of late days used on previous psets:****No. of late days used after including this pset:**

The purpose of this problem set is to reinforce the definitions of  $\text{NP}_{\text{search}}$ ,  $\text{P}_{\text{search}}$ , and  $\text{NP}_{\text{search}}$ -completeness and practice NP-completeness proofs. Note that the new **Sunday deadline** (due to the FAS policy that problem sets cannot be due after the first three days of reading period). Because of this change, this is a half-length problem set and we are also giving you 2 extra late days that you can use on this problem set (in addition to using up to 3 of your original late days that you have left over).

1. (Reductions to and from easy problems)
  - (a) Prove that if a problem  $\Pi$  is in  $\text{P}_{\text{search}}$ , then  $\Pi \leq_p \Gamma$  for all computational problems  $\Gamma$ .
  - (b) Show that all problems in  $\text{NP}_{\text{search}}$  are  $\text{NP}_{\text{search}}$ -complete if and only if  $\text{NP}_{\text{search}} \subseteq \text{P}_{\text{search}}$  (equivalently,  $\text{P} = \text{NP}$ ).
2. (Monotone SAT) A boolean formula is *monotone* if there are no negations in it. Restricting SAT to Monotone formulas makes it trivial; setting all variables to 1 is always a satisfying assignment.

However, the following variant of Monotone (3-)SAT is more interesting:

<b>Input</b> : A monotone (3-)CNF formula $\varphi(x_0, \dots, x_{n-1})$ and a number $k \in \mathbb{N}$ <b>Output</b> : A satisfying assignment $\alpha \in \{0, 1\}^n$ in which at least $k$ variables are set to 0, or $\perp$ if no such $\alpha$ exists
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**Computational Problem**  $k$ -False Monotone (3-)SAT

- (a) Prove that  $k$ -False Monotone 3-SAT is  $\text{NP}_{\text{search}}$ -complete, even when  $k = n/2$ . (Hint: reduce from 3-SAT, replacing negated variables with new ones and adding additional clauses.)
- (b) Show that if we fix  $k = 3$ , then  $k$ -False Monotone SAT is in  $\text{P}$ . (Hint: show that it suffices to consider assignments in which exactly 3 variables are set to 0.)
- (c) (\*challenge) Show that  $k$ -False Monotone 2-SAT is  $\text{NP}_{\text{search}}$ -complete. (Hint: reduce from Independent Set.)